Confinement and Radical Unification in Functional Quantum Theory of the Nonlinear Spinorfield

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The conditions are discussed under which in functional quantum theory of the nonlinear spinorfield with indefinite metric the probabilistic interpretation can be restored. This leads to a close connection of confinement, composite particles and the new subquark models of matter.

The energy-mass equivalence of Einstein enables us to assume that all physical forces and particles have a common origin. Hence the problem of high energy physics is to find out a unified dynamical law which governs the motions and reactions of matter and which allows the derivation of the hierarchy of forces and particles. In the past decade the main effort in high energy physics was devoted to the development of unified gauge quantum field theories. Such theories, for instance the grand unification approach, in principle incorporate the hierarchy of forces into one dynamical law, i. e. in this case into a basic Lagrangean functional. However, the prize for the unification of forces is an increasing number of basic fermions and gauge bosons. In addition, the proliferation of "basic" particles is continued by the increase of fermion generations which have to be taken into account. Hence it is obvious that new ideas and models must be used in order to master the unification problem.

One essential step towards a more radical solution of this problem is the idea of the fusion of integer spin particles, i. e. bosons from elementary spin half fermions. This idea was presumably inaugurated by the group theoretical spin fusion analysis of de Broglie [1]. However, the dynamical formulation of fermion fusion was prevented by the nonrenormalizability of corresponding nonlinear fermion field equations. In the fifties Heisenberg [2] proposed to consider leptons and nucleons as fundamental particles and to obtain all other elementary and nonelementary particles by fusion of the fundamental particles, i. e. as bound states of these particles. To circumvent the nonrenormalizability of his spinor field equations, Heisenberg assumed a regularization by dipole ghosts. He identified the dipole ghosts with leptons, but the meaning of this identification remained unclear. The general development of quantum field theory was not influenced by Heisenberg's approach. Nevertheless, also in this development the search for basic particles became an important topic, although boson fusion was not seriously taken into account. Gell-Mann [3] and Zweig [4] proposed a composite structure of the nucleons due to their fusion from quarks. So in this scheme the basic particles were leptons and quarks to which in a gauge theory gauge bosons have to be added. Schwinger [5] proposed a composite structure of the nucleons resp. hadrons due to their fusion from magnetic monopoles.

In recent years there was an increasing number of attempts to reach a still more elementary level of composition at which already leptons and quarks are assumed to be built up from a few basic particles, to which in most cases gauge bosons are added in order to obtain a renormalizable field theory. Insisting on boson fusion, it is obvious that with such compositions also Heisenberg's idea is renewed with the only difference that now the composition starts at a more elementary level. Formulating the new models by means of nonlinear spinorfield equations, also these equations can be or have to be regularized by dipole ghosts. In contrast to the original Heisenberg approach these new models provide a strong support for the dipole ghost regularization which was not known to Heisenberg. As the basic fermions of the new models have to be partly or completely unobservable, they must be confined, i. e. also theoretically be made unobservable. They share this confinement condition with the dipole ghosts which must be confined in order to maintain a probabilistic

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interpretation of the theory. It is therefore reasonable to identify the invisible basic fermions with dipole ghosts. For a consistent formulation of a corresponding spinorfield theory it turns out that higher order field equations are an appropriate starting point. A formulation of a lepton-quark model was given by Stumpf [6].

The regularizing properties of higher order field equations were first recognized by Mie [7]. Rigorous formulations were given by Bopp [8] and Podolski [9] at the classical level. Pais and Uhlenbeck [10] started an investigation of higher order field equations at the quantum level. Applying the usual canonical quantization procedure to such equations they obtained an indefinite metric state space. Numerous attempts were made to work with such indefinite metric field theories. A review of the older literature is given by Nagy [11] and Nakanishi [12] where the indefinite metric arising from higher order field equations is discussed. Some recent papers on this topic were published for instance by Palmer and Takahashi [13], Marx [14], Rabuffo and Vitiello [15], Narnhofer and Thirring [16], Ragiadakos [17], Barut and Crawford [18], Mintchev [19].

In addition, Nakanishi [12] showed that even in ordinary quantum field theories, including gauge theories, indefinite metric appears as soon as one leaves the point-particle picture and tries to calculate bound states. Then by the peculiarity of relativistic many particle equations negative norm states necessarily appear. Hence to work with indefinite metric is an inevitable task for any approach to derive composite particles in quantum field theories. The ordinary quantum field theoretic calculation methods are not suitable to master this problem. Therefore Stumpf and coworkers, cf. Stumpf [20], developed a calculation scheme called functional quantum theory which is defined to be a map of the ordinary state space of a quantized field into a corresponding functional state space. This functional state space allows a relativistic invariant representation of the states of a quantized field and the explicit relativistic invariant calculation of these states and their scalarproducts and is therefore especially well suited for the treatment of indefinite metric state spaces.

Having prepared an appropriate mathematical apparatus we are faced with the main problem of the theory, namely the elimination of the unwanted negative norm states. As these states, resp. their operators, effect the regularization of the theory and have, at least partly, to be identified with the basic invisible fermions, they ought not completely be eliminated, rather they must be eliminated only asymptotically in the S-matric (confinement). This procedure is called unitarization. Many proposals have been made for unitarization, cf. Nagy [11]. Most of them lack any physical interpretation and are rough actions which destroy the analyticity properties of the unitarized S-matrix. The most gentle prescription seems to be the good ghost admixture introduced by Heisenberg [2] for the Lee-model and formulated by Stumpf and Scheerer [21] for the relativistic case in functional quantum theory, the finite part description of Heisenberg [2] and the shadow-state prescription of Sudarshan [22]. For the case of dipole ghosts these three methods are equivalent as can be seen explicitly from the Lee-model calculations of Heisenberg [2]. Both Heisenberg and Sudarshan demonstrated their method practically only for nonrelativistic problems. In particular Sudarshan and coworkers investigated the consequences with respect to the analyticity of the S-matrix and emphasized that no severe violation of the causality condition has to be expected, cf. Nelson [23], Nelson and Sudarshan [24], Richard [25], Rechenberg and Sudarshan [26], Chiang [27], Sethia, Gupta and Shastry [28], Chiang [29], Chiang and Deo [30]. Without reference to other authors also Blaha [34] used this procedure. Later Newton [31] and Stapp [32] critisized these results so that a decisive answer with respect to the causality violating effects of this method cannot be given. Even if the arguments against this method can be invalidated, presumably a complete affirmative answer will never be found as its application depends on the special calculation which has to be performed. On the other hand, this method provides a very simple way to get rid of the ghost states asymptotically. In a forthcoming paper by Stumpf and Hornung, it will be shown that this method can also be applied to relativistic functional quantum theory. Hence it is reasonable to look for a general principle which avoids observable causality violations from the beginning.

Obviously such violations of causality can be avoided if the ghosts regularize the field expressions, but are not allowed to take part in the interaction of the physical particles. This implies that the transmitters of the interactions between physical particles must be themselves physical particles. As a consequence we conclude that, if at all, any physical particle must occur as a bound state of invisible particles. Then all causality violating effects occur in the interior of physical particles but will never be set free. It remains an open question whether from the structure of the physical particles such internal causality violations can be detected.

Another important proposal to realize quark confinement which can be transferred into the functional quantum theory of the nonlinear spinorfield, is concerned with the gluon propagators in gauge theories. It has been observed that gluon propagators belonging to higher order field equations of the gluons lead in nonrelativistic approximations to confining harmonic forces between the quarks. Johnson [33] used the third order derivative of the Yukawa potential $e^{-\varkappa r}/r$ with respect to $\varkappa$ as the interaction potential of two quarks. As the Yukawa potential is the nonrelativistic limit of the propagator $(p^2-\varkappa^2)^{-1}$ in ordinary space, a third power Klein-Gordon operator $(\Box-\varkappa^2)^3$ as the wave equation has to be assumed for the gluons. Later on, Blaha [34] showed that a gluon propagator p^{-4} leads to confining forces. Thus, in this case a second power d'Alembert operator $\Box^2$ as the wave equation has to be assumed. These possibilities were further investigated by Blaha [35], Bender, Mandula and Guralnik [36], Müller-Kirsten [37], Narnhofer and Thirring [16], Ragiadakos [38], Mintchev [19].

For a self-interacting spinorfield the gluon propagator has to be replaced by the spinorfield propagator of the ghost particles themselves. It is now remarkable that this construction leads to the same conditions as are required in the case of the application of the shadow state formalism, namely the distinction of an invisible subdynamics and the formation of an observable bound state dynamics. Therefore, with respect to both approaches the following program seems to be meaningful:

i) the fundamental dynamical law is given by a nonlinear spinorfield equation for dipole ghosts or higher order ghosts alone,

ii) in the corresponding functional quantum theory at the level of the ghost dynamics the Heisenberg-Sudarshan procedure or the confining ghost propagator properties have to be applied,

iii) the reactions of observable physical particles have to be derived from the functional quantum theory of the ghosts by rearrangement of the basic equations into functional equations for bound state formation and interaction.

The transformation of the fundamental unobservable dynamics into a bound state dynamics is a task which cannot be solved generally. Rather this problem has to be studied sector by sector. Then for the propagators of the bound states the usual Feynman prescription has to be applied in order to obtain analytical correct S-matrix elements. In contrast to the ordinary interaction representation of point particles, the propagators of the physical particles being bound states carry with themselves the structure of the bound state, i. e. in contrast to the point particles which are undressed, the physical particles derived in this way are dressed particles. This implies that even at the level of physical particles due to the dressing no divergencies in Feynman diagrams will occur.

These principles which are meant to minimize the observable effects of ghost confinement can be considered as an approach to radical unification. They correspond in a remarkable way to the new models of Harari [39], Shupe [40], Taylor [41], Terazawa [42], Casalbuoni and Gatto [43], Akama [44]. The Physical Review article of Terazawa contains numerous references to earlier papers on this topic. These models also use only invisible basic fermions (and bosons). A decision whether these approaches formulated in the framework of functional quantum theory are promising or not cannot be made by a general deduction, but requires the realization of the program i), ii) and iii). In contrast to the nonlinear spinorfield equation for a lepton-quark model which reads for one lepton-quark pair with $\delta := \partial^\varrho \gamma_\varrho$

$$(\delta+\mu)^2(\delta+m)\psi = V\,\psi\bar{\psi}\psi\,,$$

where $(\delta+m)$ describes the physical particle and $(\delta+\mu)^2$ the dipole ghost particle, one has now to treat equations of the type

$$(\delta+\mu)^2\psi = V[\psi\bar{\psi}]$$

or

$$(\delta+\mu')^2(\delta+\mu)^2\psi = V[\psi\bar{\psi}],$$

where $V[\psi\bar{\psi}]$ means a general interaction functional which may contain higher order local field operator products. For instance as a generalization of Fermi's four-fermion interaction Lagrangean, Akama [44] proposed a nonrenormalizable six-fermion interaction Lagrangean in order to describe the basic three-particle subquark configurations which constitute lepton and quark states. Such extensions of the interaction term may be valuable for the suppression of "forbidden" subquark configurations which frequently occur in the recent subquark models. If one intends to apply the confining propagator approach, it depends on the required properties of the corresponding propagator whether still higher order equations have to be taken into account.

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